MODEL ANSWERS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | | | | | |

Candidate Number | | | | |

Thursday 16 May 2019

Afternoon | Paper Reference **8FM0-23**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
23: Further Statistics 1
(Part of options B, E, F and G)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

1. A leisure club offers a choice of one of three activities to its 150 members on a Tuesday evening. The manager believes that there may be an association between the choice of activity and the age of the member and collected the following data.

Activity / Age a years	Badminton	Bowls	Snooker	
$a < 20$	9	3	3	15
$20 \leqslant a < 40$	10	10	14	34
$40 \leqslant a < 50$	16	15	5	36
$50 \leqslant a < 60$	15	13	11	39
$a \geqslant 60$	4	19	3	26
total	54	60	36	

(a) Write down suitable hypotheses for a test of the manager's belief.

(1)

The manager calculated expected frequencies to use in the test.

(b) Calculate the expected frequency of members aged 60 or over who choose snooker, used by the manager.

(1)

(c) Explain why there are 6 degrees of freedom used in this test.

(2)

The test statistic used to test the manager's belief is 19.583

(d) Using a 5% level of significance, complete the test of the manager's belief.

(2)

a) H_0 : The choice of activity and age of member are not associated

H_1 : The choice of activity and age of member are associated

b) total for snooker : $3 + 14 + 5 + 11 + 3 = 36$

total of $a \geq 60$: $4 + 19 + 3 = 26$

total members : 150

expected frequency : $\frac{36 \times 26}{150} = 6.24$

Question 1 continued

c) expected values:

	badminton	bowls	snooker
$a<20$	$\frac{54\times15}{150}=5.4$	$\frac{60\times15}{150}=6$	$\frac{36\times15}{150}=3.6$ ← <5
$20\leq a<40$	$\frac{54\times34}{150}=12.24$	$\frac{60\times34}{150}=13.6$	$\frac{36\times34}{150}=8.16$
$40\leq a<50$	$\frac{54\times36}{150}=12.96$	$\frac{60\times36}{150}=14.4$	$\frac{36\times36}{150}=8.64$
$50\leq a<60$	$\frac{54\times39}{150}=14.04$	$\frac{60\times39}{150}=15.6$	$\frac{60\times39}{150}$ 15.6
$a\geq60$	$\frac{54\times26}{150}=9.36$	$\frac{60\times26}{150}=10.4$	6.24

The $a<20$ and $20\leq a<40$ rows need to be merged so that the expected values are all ≥5. The table will then have 4 rows and 3 columns.

$(4-1)(3-1)=(3)(2)$

$=6$

d) critical value for 6 degrees of freedom at 5% S.L. = 12.592

$19.583>12.592$

$\therefore$ Reject H_0. The manager's belief is supported.

(Total for Question 1 is 6 marks)

2. A spinner used for a game is designed to give scores with the following probabilities

Score	1	2	3	4	6
Probability	$\frac{3}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

The spinner is spun 80 times and the results are as follows

Score	1	2	3	4	6
Frequency	15	4	12	41	8

Test, at the 10% level of significance, whether or not the spinner is giving scores as it is designed to do. Show your working and state your hypotheses clearly.

(7)

H_0 : Spinner is giving scores as it is designed to do.

H_1 : Spinner is not giving scores as it is designed to do.

Score	1	2	3	4	6
O_i	15	4	12	41	8
E_i	$80\times\frac{3}{10}=24$	$80\times\frac{1}{10}=8$	8	$80\times\frac{2}{5}=32$	8
$\frac{(O_i-E_i)^2}{E_i}$	3.375	2	2	$\frac{81}{32}$	0

$\Sigma\frac{(O_i-E_i)^2}{E_i} = 9.90625$

≈ 9.91

$V = 5 - 1$

$= 4$

critical value at 10% S.L. for 4 degrees of freedom = 7.779

$9.91 > 7.779$

$\therefore$ Reject H_0. The spinner is not giving scores as it is designed to.

Question 2 continued

(Total for Question 2 is 7 marks)

3. Andreia's secretary makes random errors in his work at an average rate of 1.7 errors every 100 words.

(a) Find the probability that the secretary makes fewer than 2 errors in the next 100-word piece of work.

(2)

Andreia asks the secretary to produce a 250-word article for a magazine.

(b) Find the probability that there are exactly 5 errors in this article.

(2)

Andreia offers the secretary a choice of one of two bonus schemes, based on a random sample of 40 pieces of work each consisting of 100 words.

In scheme **A** the secretary will receive the bonus if more than 10 of the 40 pieces of work contain no errors.

In scheme **B** the bonus is awarded if the total number of errors in all 40 pieces of work is fewer than 56

(c) Showing your calculations clearly, explain which bonus scheme you would advise the secretary to choose.

(5)

Following the bonus scheme, Andreia randomly selects a single 500-word piece of work from the secretary to test if there is any evidence that the secretary's rate of errors has decreased.

(d) Stating your hypotheses clearly and using a 5% level of significance, find the critical region for this test.

(4)

a) let x = no. of errors in 100 words

$x \sim Po(1.7)$

$P(X < 2) = P(X \leq 1)$

$= e^{-1.7}\left(\frac{1.7^1}{1!}\right) + e^{-1.7}\left(\frac{1.7^0}{0!}\right)$

$= 0.31056 + 0.18268$

$= 0.49324$

≈ 0.493 (3sf)

b) Y = no. of errors in 250 words

mean $= 1.7 \times \frac{250}{100}$

$= 4.25$

$Y \sim Po(4.25)$

$P(Y = 5) = e^{-4.25}\left(\frac{4.25^5}{5!}\right)$

$= 0.16482$

≈ 0.165 (3sf)

Question 3 continued

c) scheme A : $A \sim B(40, e^{-1.7})$

$P(A > 10) = 1 - P(A \leq 10)$

$= 1 - 0.90044$

$= 0.09956$

$\simeq 0.0996$ (3sf)

scheme B : $B \sim Po(1.7 \times 40)$

$B \sim Po(68)$

$P(B < 56) = P(B \leq 55)$

$= 0.06113$

$0.0996 > 0.0611$ ∴ the secretary should choose scheme A

d) $1.7 \times \frac{500}{100} = 8.5$

$H_0 : \lambda = 8.5$

$H_1 : \lambda < 8.5$

let E = no. of errors in 500 words

$E \sim Po(8.5)$

5% S.L. & one tailed test

=> from table

$P(E \leq 3) = 0.0301$

$P(E \leq 4) = 0.0744$

∴ critical region : $E \leq 3$

Question 3 continued

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Question 3 continued

(Total for Question 3 is 13 marks)

4. The discrete random variable X has probability distribution

x	-3	-1	1	2	4
$P(X = x)$	q	$\frac{7}{30}$	$\frac{7}{30}$	q	r

where q and r are probabilities.

(a) Write down, in terms of q, $P(X \leqslant 0)$

(1)

(b) Show that $E(X^2) = \frac{7}{15} + 13q + 16r$

(2)

Given that $E(X^3) = E(X^2) + E(6X)$

(c) find the value of q and the value of r

(7)

(d) Hence find $P(X^3 > X^2 + 6X)$

(4)

a) $P(X \leq 0) = P(X = -3) + P(X = -1)$

$= q + \frac{7}{30}$

b) $E(X^2) = (-3)^2(q) + (-1)^2(\frac{7}{30}) + (1)^2(\frac{7}{30}) + 2^2(q) + 4^2 r$

$= 13q + \frac{14}{30} + 16r$

$= \frac{7}{15} + 13q + 16r$

c) $E(6X) = 6E(X)$

$= 6[-3q + (\frac{-7}{30}) + \frac{7}{30} + 2q + 4r]$

$= 6(-q + 4r)$

$= -6q + 24r$

$E(X^2 + 6X) = E(X^2) + E(6X)$

$= \frac{7}{15} + 13q + 16r - 6q + 24r$

$= \frac{7}{15} + 7q + 40r$

$E(X^3) = (-3)^3 q + (-1)^3(\frac{7}{30}) + (1)^3(\frac{7}{30}) + (2)^3 q + (4)^3 r$

$= -27q + 8q + 64r$

$= -19q + 64r$

$\frac{7}{15} + 7q + 40r = -19q + 64r$

$26q - 24r + \frac{7}{15} = 0$ — ①

Question 4 continued

$q + \frac{7}{30} + \frac{7}{30} + q + r = 1$

$2q + r = \frac{16}{30}$

$r = \frac{16}{30} - 2q \quad — \quad (2)$

substitute (2) into (1)

$26q - 24\left(\frac{16}{30} - 2q\right) + \frac{7}{15} = 0$

$26q - \frac{384}{30} + 48q + \frac{7}{15} = 0$

$74q = \frac{185}{15}$

$q = \frac{1}{6}$

$r = \frac{16}{30} - 2\left(\frac{1}{6}\right)$

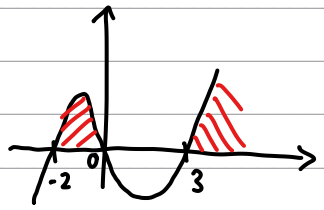
$= \frac{1}{5}$

$q = \frac{1}{6}$, $r = \frac{1}{5}$

d) $P(x^3 > x^2 + 6x) = P(x^3 - x^2 - 6x > 0)$

$= P(x(x^2 - x - 6) > 0)$

$= P(x(x-3)(x+2) > 0)$



$-2 < x < 0$ or > 3

$P(x^3 > x^2 + 6x) = P(x = -1) + P(x = 4)$

$= \frac{7}{30} + r$

$= \frac{7}{30} + \frac{1}{5}$

$= \frac{13}{30}$

Question 4 continued

(Total for Question 4 is 14 marks)

TOTAL FOR FURTHER STATISTICS 1 IS 40 MARKS